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NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

TECHNICAL NOTE

No. 1321

AN IMPROVED METHOD FOR CALCULATING THE DYNAMIC  
RESPONSE OF FLEXIBLE AIRPLANES TO GUSTS

By Abbott A. Putnam

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## SUMMARY

A set of equations based on the first mode of wing bending is presented for determining the dynamic responses of an airplane wing structure as induced by gusts. The representation of the airplane is such that the aerodynamic damping of the vibratory motion of the wing is separate from that of the vertical motion of the airplane as a whole. An easily evaluated solution of the equations for a unit-jump forcing function is also presented which may be built up to determine the response to any forcing function. A chart is included by which, for a conventional airplane, a typical forcing function for any gradient gust may be chosen quickly.

Satisfactory agreement within the limits of accuracy of the data was found in a comparison of calculated response of a semirigid test model with experimental results.

## INTRODUCTION

The dynamic response of airplane wings to gusts has become of greater interest as the size and speed of transport and bomber type airplanes has increased. Küssner (reference 1) sets forth the general equations for determining this response, but the complexity of the solution of these equations has led most investigators to make simplifying assumptions in order to reduce the equations to a form which is more readily solved. In reference 2, the assumptions made reduce the equations to those for a biplane equivalent which simulates the fundamental mode of bending of the wing of the actual airplane. A comparison of calculations with test results in reference 2 indicates that the method used yields maximum values which are in good agreement with the test results but that the calculated time histories of the responses do not agree well. The reason for the disagreement was found to be that, with the biplane equivalent, the aerodynamic-damping coefficients of the wing bending and the airplane

as a whole, which vary in relative importance, are approximated by a single constant in each damping term.

In view of the limitations of the method of reference 2, the method presented herein was devised to provide aerodynamic damping that is more representative of the actual case. Although it was derived independently from the concepts of the effects of stability and unsteady lift used in reference 2, the flexural system was considered in much the same way as it was considered by Williams and Hanson in reference 3.

Solutions of the equations of the present method for a unit-jump forcing function are also presented. These solutions may be built up into the response to arbitrary forcing functions with much less labor than when, as in reference 2, a direct solution for each forcing function is performed.

Also presented is a comparison of test results for the semi-rigid model of reference 2 with results calculated by the present method. In addition, a chart is presented to enable a typical forcing function for any gust gradient distance to be chosen quickly for a conventional airplane.

#### SYMBOLS

|                        |                                                                     |
|------------------------|---------------------------------------------------------------------|
| $t$                    | time, seconds                                                       |
| $\rho$                 | air density, slugs per cubic foot                                   |
| $\frac{dC_L}{d\alpha}$ | slope of lift curve, per radian                                     |
| $V$                    | forward velocity, feet per second                                   |
| $S$                    | gross wing area including area intercepted by fuselage, square feet |
| $S_w$                  | net wing area, square feet                                          |
| $S_f$                  | intercepted fuselage area, square feet                              |
| $y$                    | coordinate along span of wing, feet                                 |
| $m(y)$                 | mass of wing per unit span, slugs per foot                          |
| $c(y)$                 | wing chord as a function of wing spanwise station, feet             |

- $c_s$  root chord of wing, feet  
 $c_t$  tip chord of wing, feet (tapered wing)  
 $g(y)$  normalized deflection curve of wing,  $\left(\frac{\delta - \delta_f}{\delta_d}\right)$   
 $\delta_f$  absolute displacement of fuselage, positive upward, feet  
 $\delta_d$  deflection of wing tip with respect to fuselage from steady flight condition, positive upward, feet  
 $\delta$  absolute displacement of any station on wing, positive upward, feet  
 $M$  total mass of airplane, slugs  
 $M_f$  fuselage mass, slugs  
 $M_w$  wing mass, slugs  
 $M_{we}$  equivalent wing mass, slugs  
 $\lambda$  damping coefficient of airplane, pound-seconds per foot  
 $\lambda_f$  fuselage damping coefficient, pound-seconds per foot  
 $\lambda_w$  wing damping coefficient, pound-seconds per foot  
 $\lambda_{we}$  equivalent wing damping coefficient, pound-seconds per foot  
 $f_w$  natural frequency of wing, fuselage fixed, cycles per second  
 $f_{wf}$  natural frequency of wing-fuselage system about nodes, cycles per second  
 $K$  spring constant, the upward force on fuselage due to both wings when in assumed deflection shape, per unit tip deflection, pounds per foot  
 $F_w$  fraction of forcing function acting on wing  
 $F_f$  fraction of forcing function acting on fuselage  
 $Ate^{-bt}$  forcing function assumed for conventional airplane

$$A = \frac{\Delta n_a w}{te^{-bt}} \quad \text{at } bt = 1 \quad \text{or} \quad A = Wbe \Delta n_a$$

$\Delta n_a$  arbitrary load-factor increment that the airplane would experience if it had no vertical motion as it traversed the gust,  $g$

$W$  total weight of airplane, pounds

$$b = \frac{1}{t_F}$$

$t_F$  time to reach forcing function peak from entry, seconds

$t_G$  time to reach gust peak from entry, seconds

$1$  unit-jump wing forcing function ( $1 = 0$  for  $t < 0$   
 $1 = 1$  for  $t > 0$ )

$$\eta = \frac{M\lambda_{we}}{\lambda M_{we}}$$

$$\beta = \frac{M\lambda_f}{\lambda M_f}$$

$$\zeta = \frac{M\lambda_w}{\lambda M_w}$$

$$\gamma = \left(\frac{M}{\lambda}\right)^2 \frac{KM}{M_f M_{we}}$$

$Z$  real root of equation  $z^3 + (\eta + \beta)z^2 + (\gamma + \eta\beta)z + \gamma = 0$

$\psi$  real part of complex roots of foregoing equation  $\left(\psi = -\frac{Z + \eta + \beta}{2}\right)$

$\Omega$  imaginary part of complex root of equation represented by  $Z$

$$\left(\Omega = \sqrt{\frac{-\gamma}{Z} - \psi^2}\right)$$

$$\Delta = \psi - Z$$

$$\Gamma = 3Z^2 + 2(\eta + \beta)Z + (\gamma + \eta\beta)$$

$$P = \frac{F_F M}{M_F}$$

$$Q = \frac{F_F M \eta}{M_F}$$

$$E = PZ^2 + QZ + \gamma$$

$$L = \frac{F_W M}{M_{We}} - \frac{F_F M M_W}{M_F M_{We}}$$

$$N = \frac{F_W M \beta}{M_{We}} - \frac{F_F M M_W \zeta}{M_F M_{We}}$$

$$B = LZ + N$$

## METHOD OF ANALYSIS

### Equations

In the derivation of the present equations for the dynamic response of airplane wings to gusts, certain of the initial assumptions are the same as those of reference 2, namely:

- (a) The loading is symmetrical
- (b) Only the first mode of bending is important
- (c) The deflection at any point on the wing may be expressed with the necessary accuracy as  $g(y)$  times the tip deflection relative to the fuselage where  $g(y)$  is variable with span but independent of time
- (d) Damping varies linearly with vertical velocity

Instead of the biplane equivalent of reference 2, however, the airplane is considered as shown in figure 1. With this representation, the equation for the summation of the vertical forces on the fuselage under the unit-jump-type forcing function  $F_f / l$  is:

$$M_f \frac{d^2 \delta_f}{dt^2} + \lambda_f \frac{d \delta_f}{dt} - K \delta_d = F_f \quad (1)$$

and the equation for the summation of the vertical forces over the entire wing under the corresponding forcing function.  $F_w$  is:

$$2 \int_{\text{root}}^{\text{tip}} m(y) dy \frac{\partial^2 \delta}{\partial t^2} + 2 \int_{\text{root}}^{\text{tip}} \frac{\lambda_w}{S_w} c(y) dy \frac{\partial \delta}{\partial t} + K \delta_d = F_w \quad (2a)$$

When the indicated integrations are performed, equation (2a) reduces to:

$$M_w \frac{d^2 \delta_f}{dt^2} + \lambda_w \frac{d \delta_f}{dt} + M_{we} \frac{d^2 \delta_d}{dt^2} + \lambda_{we} \frac{d \delta_d}{dt} + K \delta_d = F_w \quad (2b)$$

The advantage of using the present concept of the airplane as shown in figure 1 over the biplane equivalent of reference 2 is immediately apparent when the damping terms of equations (1) and (2b) are examined. The damping of the vertical motion of the airplane as a whole is now separated from the damping of the vibratory motion of the wing, which thus eliminates the necessity for the compromise made to represent the damping when the biplane equivalent is used. This result is achieved by redefinition of the mass and mass distribution and by basing the damping of the equivalent wing mass on the vibrational velocity rather than on its absolute vertical velocity.

The solutions of equations (1) and (2b) under action of the unit-jump forcing function for the fuselage acceleration increment, wing-tip acceleration increment relative to the fuselage increment, and the wing-tip displacement relative to the fuselage with the initial conditions of steady flight, when  $\delta_f = \delta_d = \frac{d \delta_f}{dt} = \frac{d \delta_d}{dt} = 0$  are:

$$\begin{aligned} \frac{d^2 \delta_f}{dt^2} = \frac{1}{M} \left\{ \frac{E}{\Gamma} e^{\frac{\lambda}{M} Z t} + e^{\frac{\lambda}{M} \psi t} \left[ \left( P - \frac{E}{\Gamma} \right) \cos \frac{\lambda}{M} \Omega t \right. \right. \\ \left. \left. + \frac{1}{\Omega} \left( P \psi + P Z + Q + \frac{\Delta E}{\Gamma} \right) \sin \frac{\lambda}{M} \Omega t \right] \right\} \quad (3) \end{aligned}$$

$$\frac{d^2\delta_d}{dt^2} = \frac{1}{M} \left\{ \frac{ZB}{\Gamma} e^{\frac{\lambda}{M} Zt} + e^{\frac{\lambda}{M} \psi t} \left[ \left( L - \frac{ZB}{\Gamma} \right) \cos \frac{\lambda}{M} \Omega t + \left( \frac{\psi L}{\Omega} + \frac{\psi \Delta B}{\Omega \Gamma} + \frac{\Omega B}{\Gamma} \right) \sin \frac{\lambda}{M} \Omega t \right] \right\} 1 \quad (4)$$

$$\delta_d = \frac{M}{\lambda^2} \left\{ \frac{N}{\gamma} + \frac{B}{Z\Gamma} e^{\frac{\lambda}{M} Zt} + e^{\frac{\lambda}{M} \psi t} \left[ - \left( \frac{B}{Z\Gamma} + \frac{N}{\gamma} \right) \cos \frac{\lambda}{M} \Omega t + \left( \frac{\Delta B}{\Gamma \Omega Z} + \frac{\psi N}{\gamma \Omega} \right) \sin \frac{\lambda}{M} \Omega t \right] \right\} 1 \quad (5)$$

The acceleration increment of any point on the wing relative to the ground, which is of interest in certain design problems, may be found from the equation

$$\frac{\partial^2 \delta}{\partial t^2} = \frac{d^2 \delta_f}{dt^2} + g(y) \frac{d^2 \delta_d}{dt^2} \quad (6)$$

If the applied gust is assumed to be uniform across the span of the airplane so that the distribution of forcing function is similar to that of the damping of the vertical motion, that is, if

$$\frac{1}{\lambda} = \frac{F_f}{\lambda_f} = \frac{F_w}{\lambda_w}, \text{ equations (4) and (5) may be simplified to:}$$

$$\frac{d^2\delta_d}{dt^2} = \frac{L}{M} \left\{ \frac{Z^2}{\Gamma} e^{\frac{\lambda}{M} Zt} + e^{\frac{\lambda}{M} \psi t} \left[ \left( 1 - \frac{Z^2}{\Gamma} \right) \cos \frac{\lambda}{M} \Omega t + \left( \frac{\psi}{\Omega} + \frac{\psi Z \Delta}{\Omega \Gamma} + \frac{Z \Omega}{\Gamma} \right) \sin \frac{\lambda}{M} \Omega t \right] \right\} 1 \quad (7)$$

$$\delta_d = \frac{M}{\lambda^2} \frac{L}{\Gamma} \left[ e^{\frac{\lambda}{M} Zt} + e^{\frac{\lambda}{M} \psi t} \left( - \cos \frac{\lambda}{M} \Omega t + \frac{\Delta}{\Omega} \sin \frac{\lambda}{M} \Omega t \right) \right] 1 \quad (8)$$

If, in a given case, it is felt that torsion of the wing would invalidate the results, a first approximation to the response



of the first combined bending-torsion mode may be obtained by the redefinition of the spring constant, the equivalent wing mass, and the wing damping coefficient to take into account the added degree of freedom. Terms which contain the redefined constants then merely replace the terms previously used when equations (3), (4), and (5) are evaluated.

### Evaluation of Constants

Damping coefficients.- The damping coefficients are evaluated from the relation  $\lambda = 0.75 \frac{dC_L}{d\alpha} \frac{\rho}{2} SV$  where  $\frac{dC_L}{d\alpha} \frac{\rho}{2} SV$  is the term for the steady lift per unit vertical velocity increment. An effective damping factor of 0.75 is inserted to take care of the lag in lift and variable damping of the actual case. The simplifying assumption of a constant effective damping factor rather than the variable factor of the exact theory is justified by a series of tests and computations reported in reference 2. The damping coefficients are therefore

$$\lambda_w = 0.75 \frac{dC_L}{d\alpha} \frac{\rho}{2} VS_w$$

$$\lambda_f = 0.75 \frac{dC_L}{d\alpha} \frac{\rho}{2} VS_f$$

and

$$\lambda_{we} = 2 \left( 0.75 \frac{dC_L}{d\alpha} \frac{\rho}{2} V \right) \int_{\text{root}}^{\text{tip}} c(y) g(y) dy$$

The coefficient  $\lambda_{we}$  includes the variation of wing velocity relative to the fuselage in accordance with the assumed wing deflection curve. If the deflection of the wing is assumed to vary with square of the distance from the root,  $\lambda_{we}$  will equal  $\frac{\lambda_w}{4}$  for elliptical wings, and  $\frac{\lambda_w}{6} \left( \frac{c_s + 3c_t}{c_s + c_t} \right)$  for tapered wings.

Spring constant and equivalent wing mass.- When the equivalent wing mass  $M_{we}$  is determined from the definition

$$M_{we} = 2 \int_{\text{root}}^{\text{tip}} m(y) g(y) dy,$$

the spring constant  $K$  may be computed from  $M_{we}$  and either of the natural frequencies  $f_w$  or  $f_{wf}$ . If the

fuselage is fixed and no damping is present, the solution for the natural frequency of the wing leads to  $f_w = \frac{1}{2\pi} \sqrt{\frac{K}{M_{w_e}}}$ . In the case where the wing-fuselage combination is free to vibrate about the natural nodes, the solution gives the frequency  $f_{wf} = \frac{1}{2\pi} \sqrt{\frac{KM}{M_{w_e} M_f}}$ .

Forcing function.— The forcing-function ratios  $F_f$  and  $F_w$  may be evaluated from the assumption that the forcing-function forces are distributed proportionally to the damping forces; thus  $F_f = \frac{\lambda_f}{\lambda}$  and  $F_w = \frac{\lambda_w}{\lambda}$  and, although this assumption may not be exactly correct, it allows  $F_f$  and  $F_w$  to remain constant throughout the time history.

Solution for roots of auxiliary equation.— In order to determine the solution of the differential equations, the cubic

$$z^3 + (\eta + \beta)z^2 + (\gamma + \eta\beta)z + \gamma = 0$$

must be solved for the three roots,  $Z$ ,  $\psi + i\Omega$ , and  $\psi - i\Omega$ . For typical cases, the value of  $Z$  has been found to be between  $-1.000$  and  $-1.020$ , and can be determined with sufficient accuracy by straight line interpolation of the value of the cubic at  $z = -1.000$  and  $z = -1.010$ . The value of  $\psi$  is then  $-\frac{Z + \eta + \beta}{2}$

and  $\Omega$  is  $\sqrt{-\frac{\gamma}{Z} - \psi^2}$ .

#### Determination of Response to Gusts

In order to determine the response of the flexible airplane to a given gust, a forcing function representing the forces on the airplane when considered to be rigid is combined with the response of the flexible airplane to the unit-jump forcing function by use of Duhamel's integral. Since equations (1) and (2b) contain terms representing the damping of the vertical motion of the airplane as a whole, the forcing function representing the gust need only contain the effects of the imposed gust and of the pitching stability of the airplane. Although the function may be computed for each gust shape and for each airplane, it was found (reference 2) that for most conventional airplanes the forcing function could be represented by a curve of the general form  $Ate^{-bt}$ . In order that, for a given airplane, the function represent a specific gust shape, a relation

must be established between it and the shape of the gust. It has been found that if a rigid airplane model is flown through a gust of a standard shape (that is, one having a linear increase of gust velocity to a maximum value which is maintained), reaching the maximum at the time  $t_G$  after entering the gust, the acceleration record of the airplane will show a maximum at approximately the time  $t_G$  (reference 4). The desired relation is, therefore, one for which the maximum acceleration of the flexible airplane, if considered rigid, occurs at  $t_G$ . The equation containing this relation

$$\left(\frac{\lambda}{M} t_F\right)^2 \left(\frac{M}{\lambda t_F} - 1\right) e^{\frac{\lambda}{M} t_G} = 2 \frac{\lambda}{M} t_F + \frac{\frac{\lambda}{M} t_G}{\frac{\lambda}{M} t_F} - 1 - \frac{\lambda}{M} t_G \quad (10)$$

may be obtained from the equation for the reaction of the rigid airplane to the impressed forcing function

$$M \frac{d^2 \delta_F}{dt^2} + \lambda \frac{d \delta_F}{dt} = A t e^{-bt} \quad (11)$$

by solving for the acceleration and determining the time  $t_G$  to peak acceleration. If the dimensionless values  $\frac{\lambda}{M} t_F$  and  $\frac{\lambda}{M} t_G$  are used, the solutions of equation (10) may be expressed as the single curve in figure 2. Thus, values of  $t_F$ , which are the inverse of  $b$ , may be obtained directly from  $t_G$ .

#### APPLICATION OF METHOD

Since the absolute values of the response of an airplane to a known true gust velocity cannot be determined at present, the arbitrary multiplying constant  $A$  is used in the forcing-function equation. By application of the methods outlined in reference 2, however, the ratios of the dynamic responses to the responses under so-called static conditions may be computed. These ratios, which are independent of the arbitrary constant, are then used as multiplying factors to the corresponding stresses and acceleration increments determined by normal static design procedure to determine these responses under dynamic conditions.

## COMPARISON OF THEORY AND TESTS

As a measure of the accuracy which might be expected from this theory, responses have been computed for the semirigid model reported in reference 2 and are compared herein with test results. The quality of the test results and the factors which may cause differences between theory and test results have been discussed in considerable detail in reference 2 and will not be mentioned further.

A linear deflection curve was used for  $g(y)$ , the deflection of the wing, and it was assumed that the lift and damping forces on the fuselage were negligible and could be set equal to zero. The spring constants for the two frequencies tested and other coefficients were evaluated from the original test data but may be obtained from the equivalent values given in reference 2.

In figure 3 are shown the theoretical curves for the variation with gust gradient distance of the ratio of the maximum wing tip deflection to the maximum fuselage acceleration for two wing frequencies, and the test results are plotted thereon for comparison. It should be noted that the straight horizontal lines obtained are for the test hinged wing with no fuselage lift and damping. In the general case, the line would have some curvature, the direction and magnitude of which would depend on all the airplane and gust-shape parameters.

At the higher frequencies in figure 3 there seems to be a tendency for the curves to fall a little low for both wings. This result could indicate that the actual frequency of the wings was not quite so high as 26.1 cycles per second but, because of the scatter of the points, nothing definite can be concluded. Consider only the left wing, which according to reference 2 produced the set of wing data more likely to be correct. The points are noted to fit a straight horizontal line. In the case of curves for the lower wing frequency, the scatter appears larger but, on a percentage basis, is not. The tendency is for the data for the right wing to fall high and that for the left wing to fall low. Since it appears (reference 2) that the right wing hinge tended to buckle, the indicated differences would be expected. Again, no definite conclusions can be drawn, although the straight horizontal line seems to be a good fit to the test points.

Sample test wing-tip-deflection curves are compared with the calculated curves for three gust gradient distances for a wing frequency of 13.5 cycles per second in figure 4, and for a wing frequency of 26.1 cycles per second in figure 5. It will be noted that in all the curves the data are plotted as the ratio  $\delta_d/\delta_{d_{max}}$

so that prediction of the magnitude of the forcing function from the gust velocity is not necessary.

The agreement between the shapes of the calculated and the experimental curves is satisfactory in every case, especially when it is considered that the general forcing function  $Ate^{-bt}$  was used in the calculations and that many unknown factors can cause the test results to deviate from those for the assumed condition. The timing of the peaks, which depends largely on the natural frequency of the wings, is good, except for the sharp-edge gust, where the use of an average effective 4.6-chord gradient distance, as suggested in reference 2, for computing all responses for the test model would affect the results. The amount of damping of the various curves, however, seems to be somewhat underestimated in the computation. Since the fuselage-acceleration curves of the tests show even more damping present than the calculations, it appears that a small amount of fuselage damping should be included. In the case of the conventional airplane, this fuselage damping has been allowed for in the theory by use of the intercepted wing area, and deviations between theory and test from this cause should be minor.

Comparison of the results shown in figures 4 and 5 with similar results given in figures 13 and 14 of reference 2 indicates that the method of accounting for the damping distribution in the present paper yields calculated time histories of results that are more in conformity with actual conditions. Although the two methods yield about the same results for the maximum responses to single gusts, the time histories determined for single gusts by the present method are believed to be more accurate for use in building up the reactions to repeated gusts.

#### CONCLUDING REMARKS

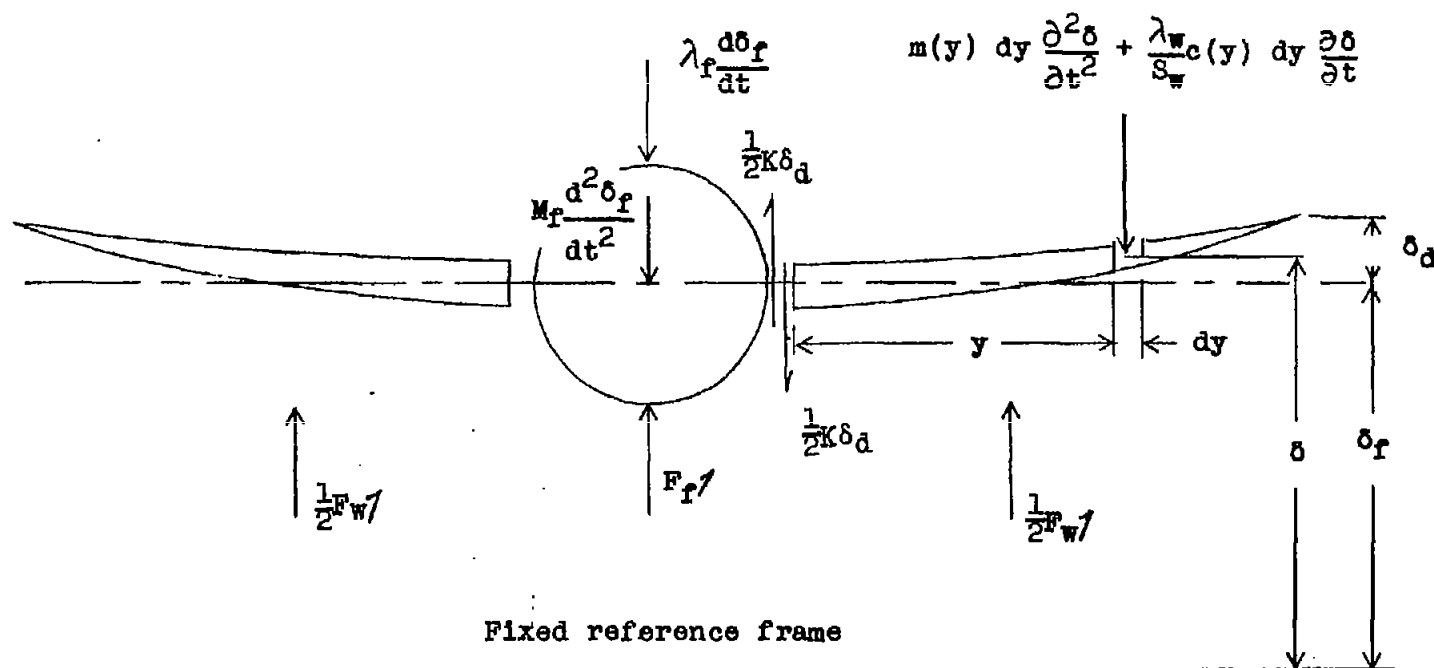
A comparison of test results on a semirigid model with the results of calculations made by the method presented indicates that an improvement has been made in the calculation of the time histories of wing deflection and acceleration increments without impairing the accuracy of the calculation of maximum values. In addition, the solution of the basic equations for the unit-jump forcing function, together with the presentation of a chart for determining a suitable forcing function for a given gust, has materially reduced the time necessary for making a set of

calculations of the dynamic response of the wings of a conventional airplane to a gust.

Langley Memorial Aeronautical Laboratory  
National Advisory Committee for Aeronautics  
Langley Field, Va. January 22, 1947

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3. Williams, D., and Hanson, J.: Gust Loads on Tails and Wings. R. & M. No. 1823, British A.R.C., 1937.
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Figure 1.- Loads on flexible airplane.

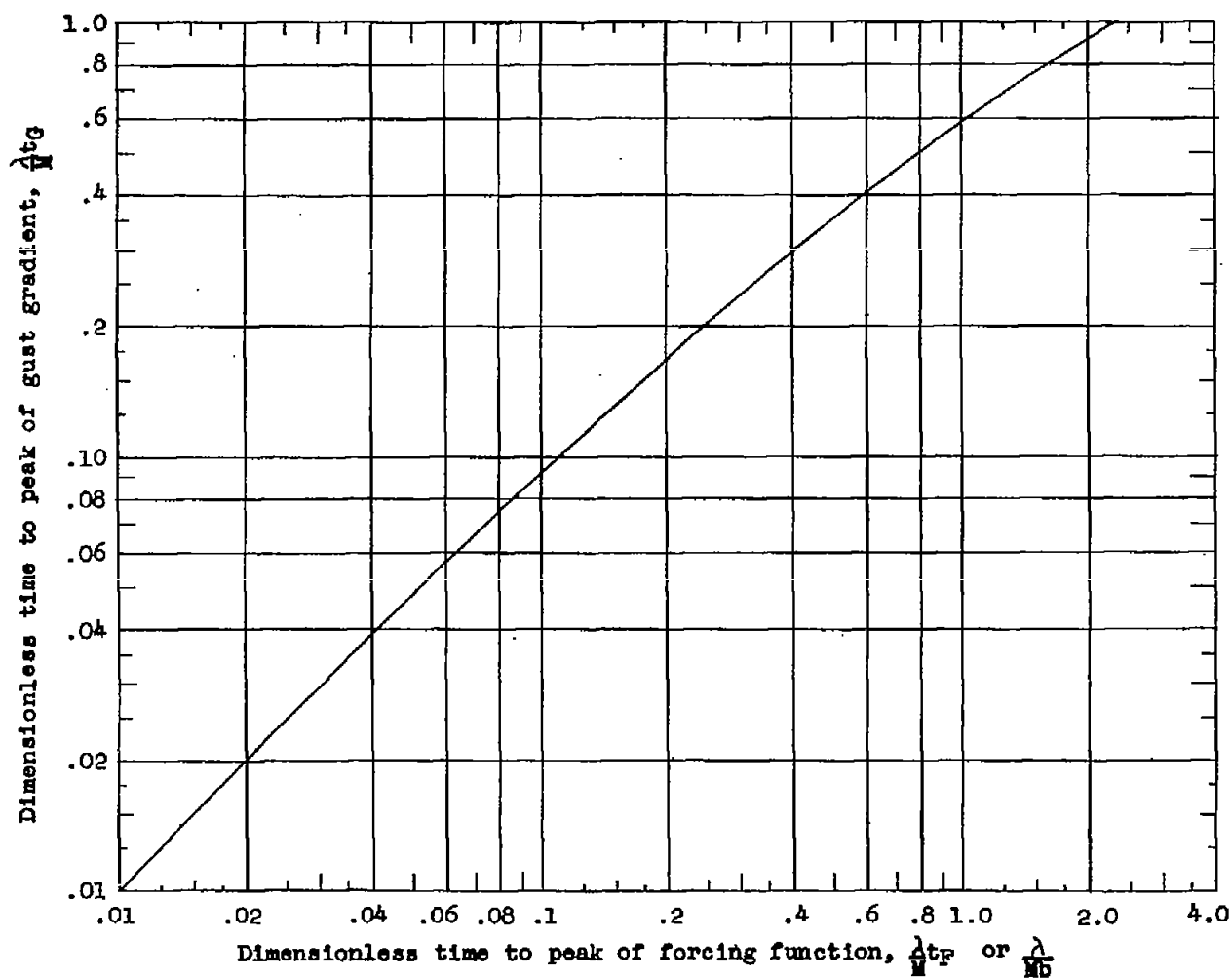
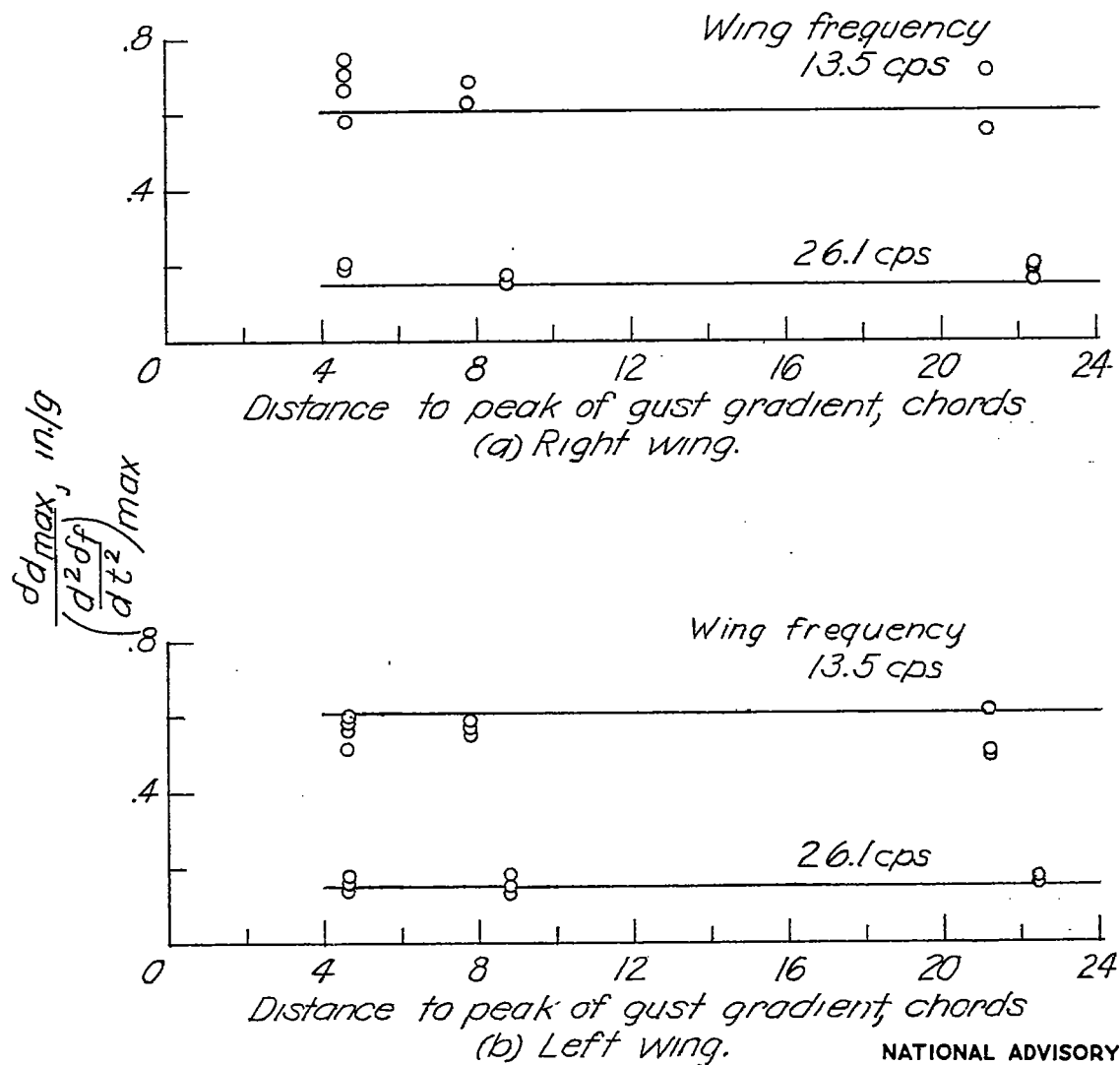


Figure 2.- Chart to determine value of  $b$  for forcing function,  $\Delta t e^{-bt}$ .

$$\left(\frac{\lambda t_F}{M}\right)^2 \left(\frac{M}{\lambda t_F} - 1\right) e^{\frac{\lambda t_G}{M}} = 2 \frac{\lambda}{M} t_F + \frac{\frac{\lambda}{M} t_G}{\frac{\lambda}{M} t_F} - 1 - \frac{\lambda t_G}{M}$$

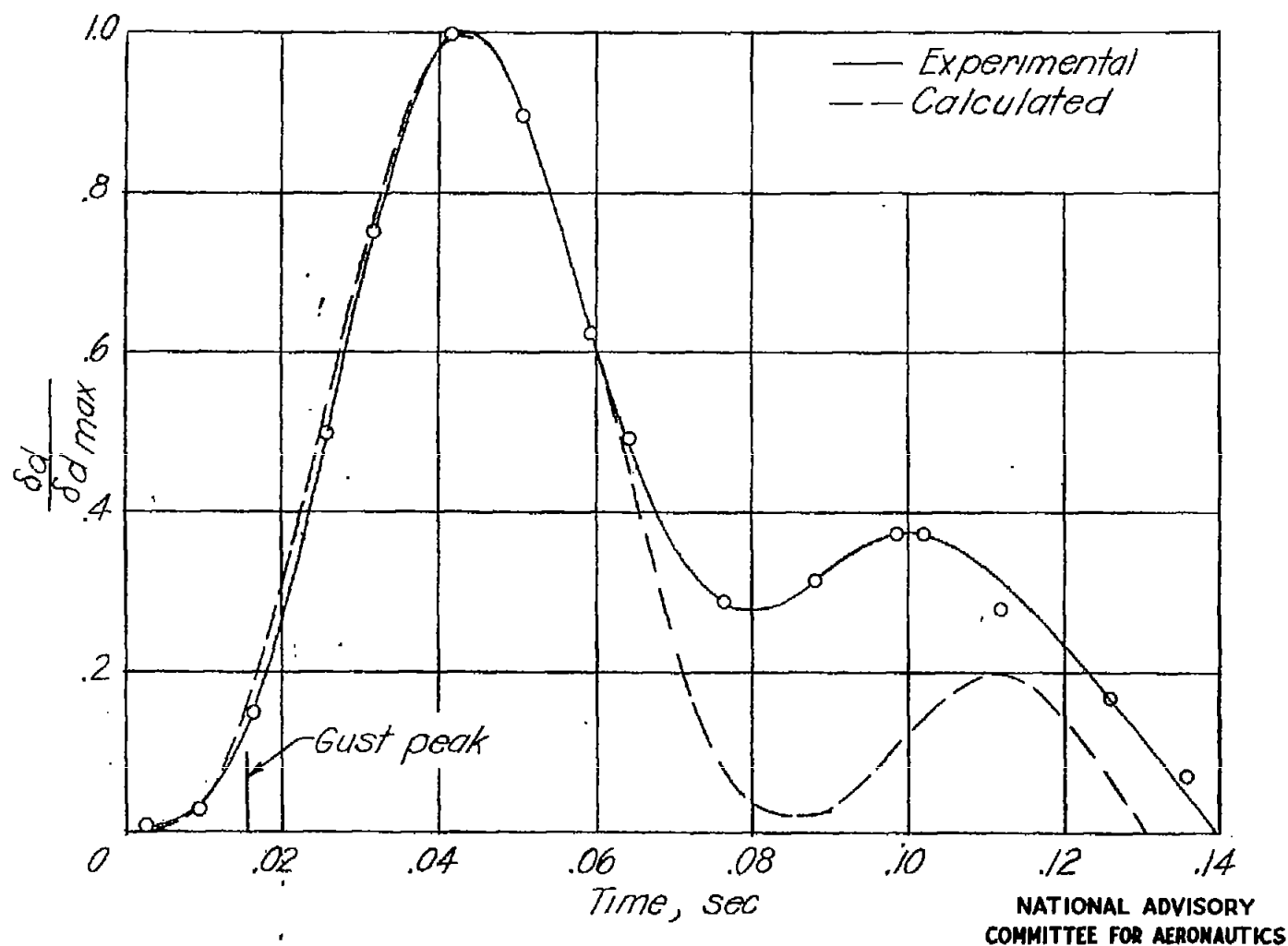
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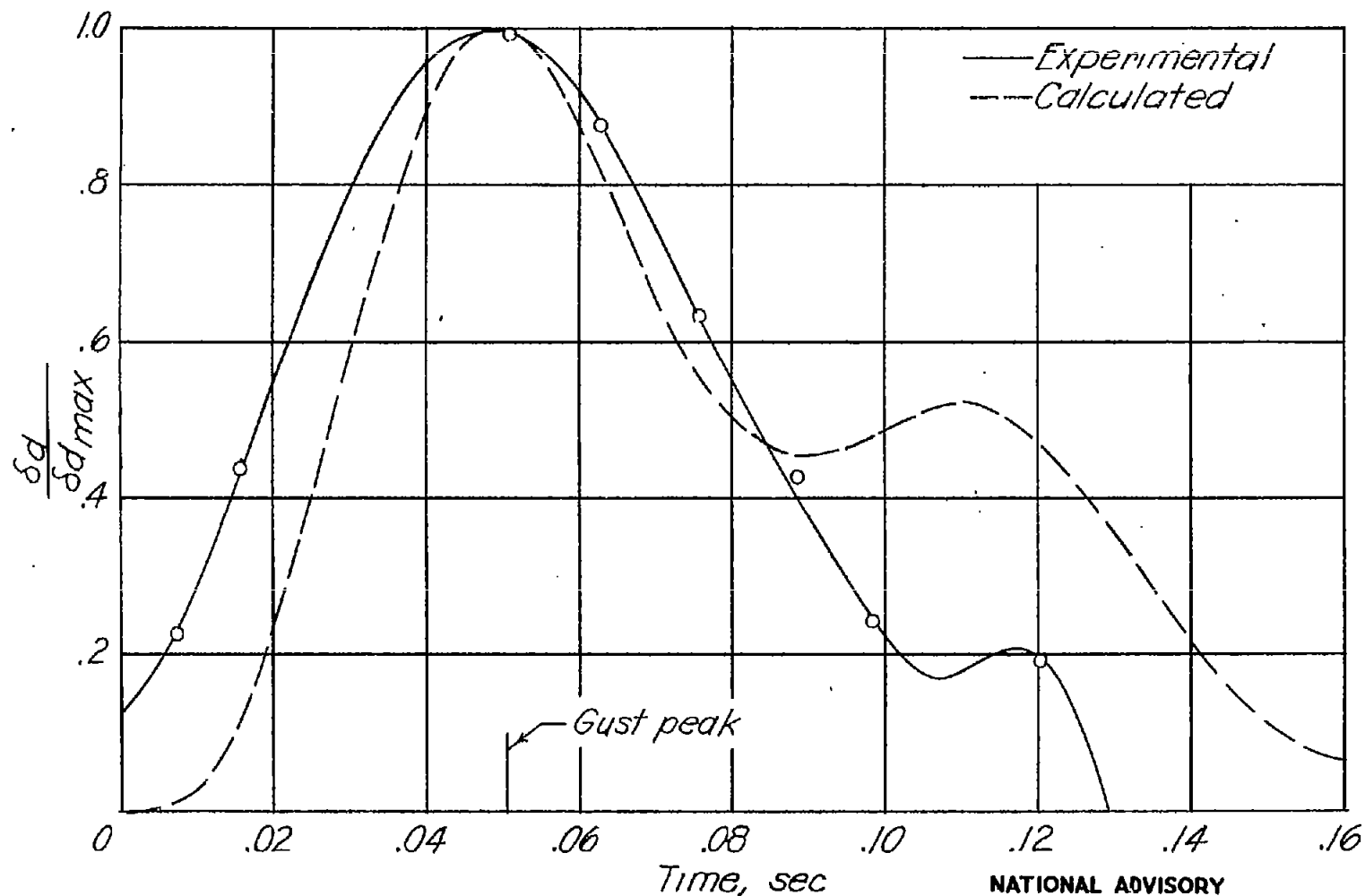


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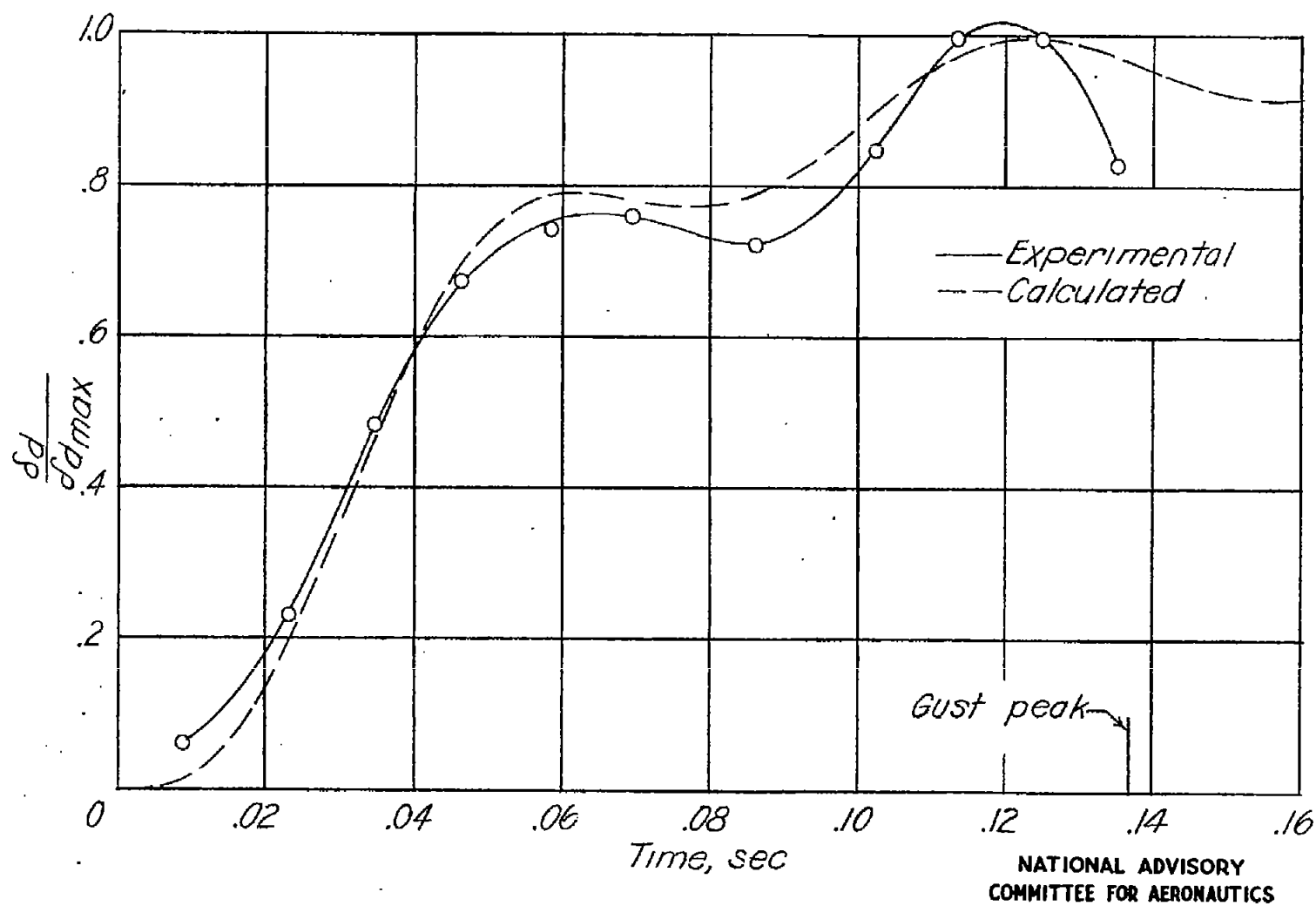
Figure 3.- Comparison of calculated and experimental values of the ratio of maximum wing-tip deflection to maximum fuselage acceleration for various gust gradient distances.



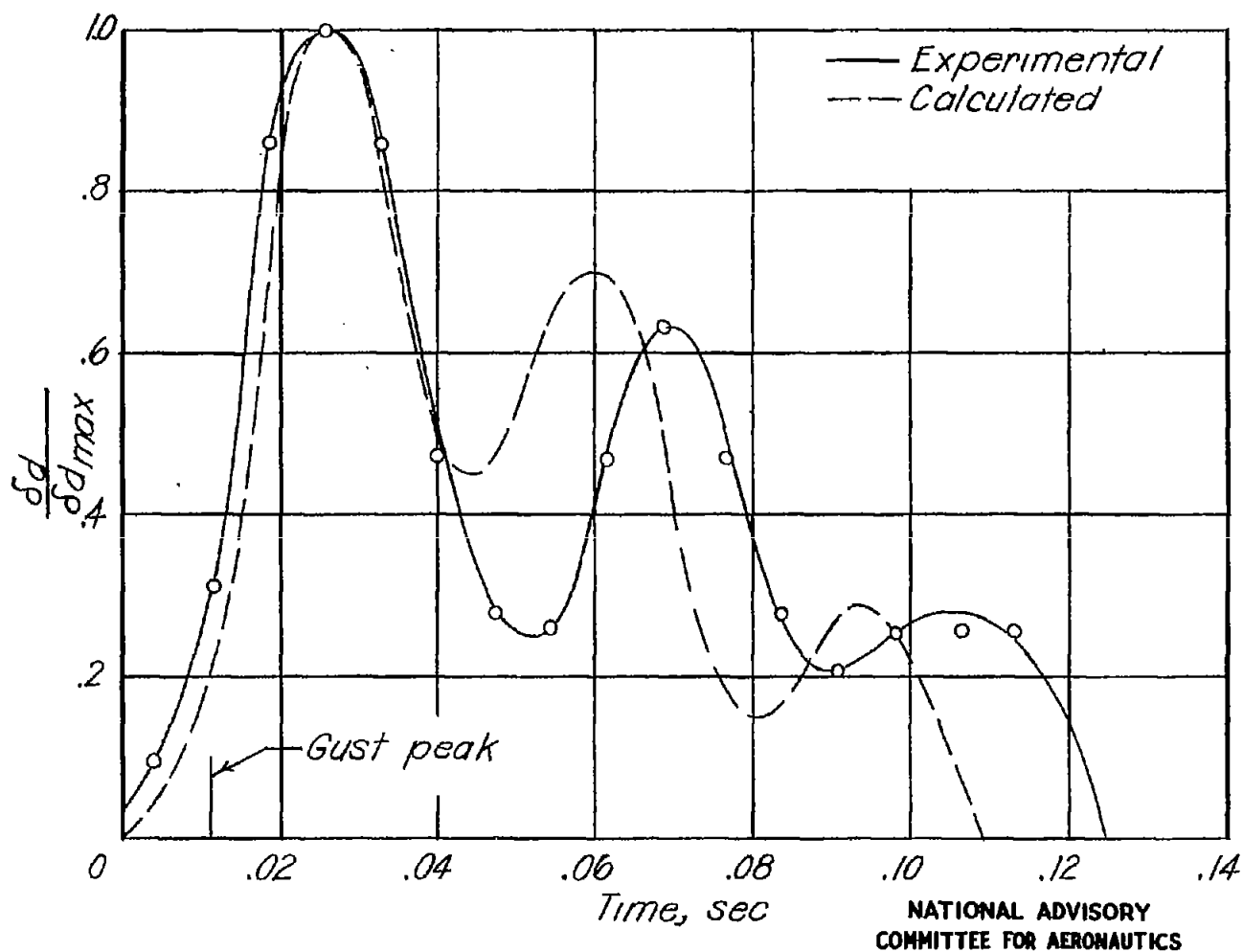
(a) Sharp-edge gust with 2.4-chord gradient distance.  
 Figure 4.—Comparison of experimental and calculated wing  
 deflections due to a gust.  $fw_f = 13.5$  cps.



(b) Gust with 7.8-chord gradient distance.  
Figure 4.- Continued.



(c) Gust with 21.2-chord gradient distance.  
Figure 4.- Concluded.



(a) Sharp-edge gust with 1.8-chord gradient distance.  
 Figure 5.- Comparison of experimental and calculated wing deflections due to a gust.  $f_{wf} = 13.5$  cps.

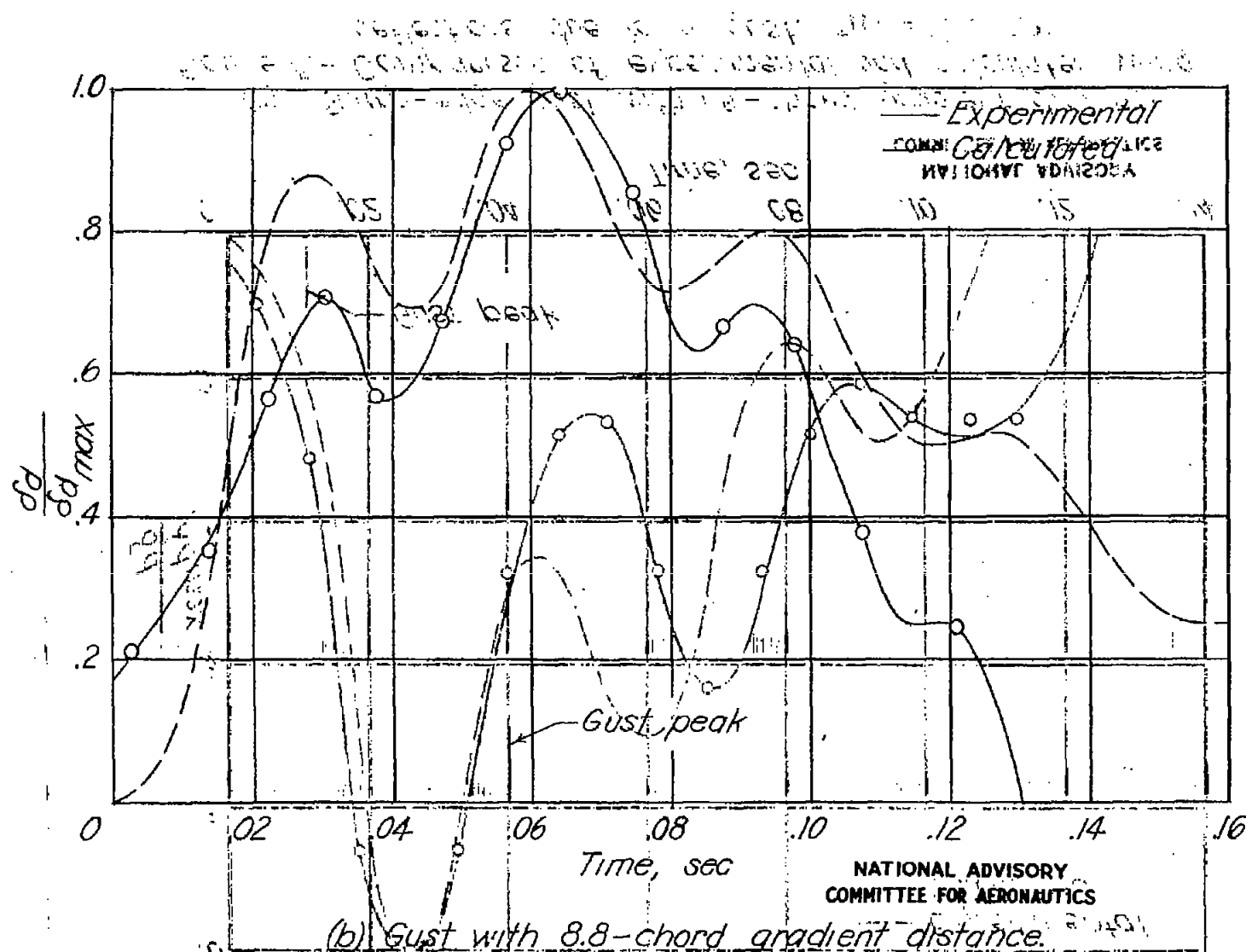
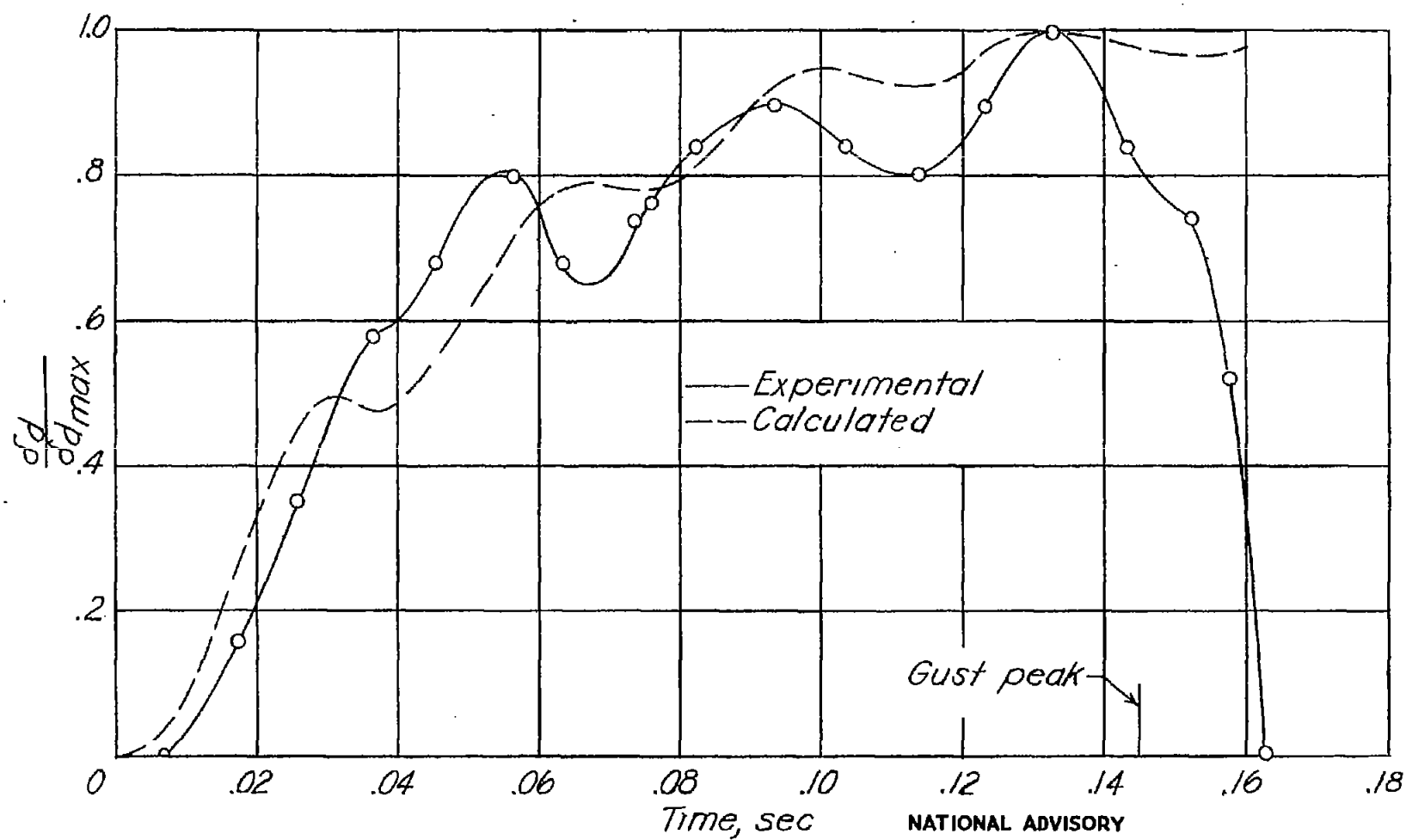


Figure 5.- Continued.



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(c) Gust with 22.4 - chord gradient distance.  
Figure 5.- Concluded.